

# Two point functions of BCFT

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## Abstract

Boundary conformal field theory (BCFT) is a conformal field theory (CFT) on manifolds with a boundary. We can use conformal symmetry to constrain correlation function of conformal invariant fields. We compute two-point function of conformal invariant fields which live in semi-infinite space. For a situation with a boundary condition in surface  $z = \bar{z}$ , our result agrees with what have been obtained recently using AdS/BCFT result.

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# 1 Introduction

The AdS/CFT correspondence [1, 2, 3], which enables us to study conformal field theory and non-perturbative quantum gravity at the same time, has been considered during the past decade. Holographic dual of a conformal field theory defined in domain with a boundary was proposed in [4]. The main idea of AdS/BCFT correspondence was started with asymptotically AdS geometry with Neumann boundary condition on the metric as one approaches to the boundary [4]. The action of this theory is given by Einstein-Hilbert action with a negative cosmological constant which is added by boundary term [4, 5, 6].

$$S = \frac{1}{16\pi G} \int_M d^3x \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G} \int_{\partial M} d^2x \sqrt{-h} K + S_M \quad (1)$$

where  $g$  and  $h$  are the 3D bulk and 2D boundary metrics. The second term is the Gibbons-Hawking boundary term [7], which is given by  $K = h^{ab} K_{ab}$ . Extrinsic curvature  $K_{ab}$  defined by  $K_{ab} = \nabla_a n_b$  where  $n$  is the unit vector normal to  $\partial M$ .  $S_M$  is action of some matter fields on the boundary  $\partial M$ . The geometry is modified by imposing two different boundary conditions on the metric. By this method the boundary is divided into two parts  $\partial M = N \cup Q$  where  $\partial Q = \partial N$  [4]. The metric has Neumann boundary condition on  $Q$  and Dirichlet boundary condition on  $N$ . The variation of the action (1) with respect to boundary metric  $h_{ab}$  leads to

$$\delta S = \frac{1}{16\pi G} \int_Q \sqrt{-h} (K_{ab} \delta h^{ab} - K h_{ab} \delta h^{ab} - 8\pi G T_{ab} \delta h^{ab}) d^2x \quad (2)$$

where

$$T_{ab} = -\frac{2}{\sqrt{-h}} \frac{\delta S_M}{\delta h_{ab}} \quad (3)$$

Neumann boundary condition was imposed by setting coefficients of  $\delta h_{ab}$  to zero, so this equation is obtained

$$K_{ab} - h_{ab} K = 8\pi G T_{ab} \quad (4)$$

If the boundary matter Lagrangian is a constant, the action has the following form

$$S = \frac{1}{16\pi G} \int_M d^3x \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G} \int_{\partial M} d^2x \sqrt{-h} (K - T) + S_M \quad (5)$$

$T$  is interpreted as the tension of the boundary surface  $Q$ . With this boundary condition ( Neumann boundary condition on  $Q$  and Dirichlet boundary condition on  $N$ ) the AdS geometry is divided into two parts and the gravitational theory lives in one part of this space. This modified geometry could provide a holographic dual for BCFT [4]. Boundary conformal field theory (BCFT) defined in domains with a boundary [8]. When CFT lives in semi-infinite space, one sector of conformal group is removed. For example, if we have a boundary condition on surface  $z = \bar{z}$  *i.e.*  $(x+t = x-t)$  or  $t = 0$ , time-translation, Boost and time-spacial conformal transformation are removed. So, two-point function in this situation is completely different from situation without boundary condition (free space). In this paper we study two point correlation functions of BCFT. By using some methods in conformal field theory [9], we calculated two-point function in semi-infinite spaces and our results agree with quantum gravity results [6]. The paper is organized as follow. In section (2) we build the representations of infinite conformal algebra in two dimensions [9]. In section (3) we calculate two-point function in free space. Then in section (4) we extended this calculation to the space with boundary conditions. Finally in section (5), we close by some concluding remarks.

## 2 Representations of 2D conformal algebra

In this work we calculated correlation function of conformal invariant fields which live in semi-infinite two-dimensional space-time. Firstly, in this section we build the representations of infinite conformal algebra in two dimensions. Conformal algebra is constructed by two copies of non-centrally Virasoro (Witt) algebra [10]

$$\begin{aligned} [L_n, L_m] &= (n - m)L_{n+m} \\ [\bar{L}_n, \bar{L}_m] &= (n - m)\bar{L}_{n+m} \\ [L_n, \bar{L}_m] &= 0 \end{aligned} \tag{6}$$

where  $L_{-1}$  ( $\bar{L}_{-1}$ ),  $L_0$  ( $\bar{L}_0$ ) and  $L_1$  ( $\bar{L}_1$ ) are translation, dilatation and spacial conformal transformation (SCT) generators in  $z$  ( $\bar{z}$ ) direction respectively. The representation of conformal algebra is built by considering operators which are labeled by dilatations eigenvalues. Local operators are given by

$$\mathcal{O}(z, \bar{z}) = U \mathcal{O}(0) U^{-1} \quad \text{where} \quad U = e^{zL_{-1} + \bar{z}\bar{L}_{-1}} \tag{7}$$

From Eq.(6) we note that  $[L_0, \bar{L}_0] = 0$  and  $[L_0, L_n] \sim L_n$  ( $[\bar{L}_0, \bar{L}_n] \sim \bar{L}_n$ ), so the representations of conformal algebra should be labeled by eigenvalues of these two operators ( $L_0$  and  $\bar{L}_0$ ). We introduce local operators which are simultaneous eigenstates of  $L_0$  and  $\bar{L}_0$ .

$$[L_0, \mathcal{O}] = h\mathcal{O} \qquad [\bar{L}_0, \mathcal{O}] = \bar{h}\mathcal{O} \qquad (8)$$

where  $h$  ( $\bar{h}$ ) is left (right) conformal weight ( $h = \frac{\Delta+s}{2}, \bar{h} = \frac{\Delta-s}{2}$  where  $\Delta$  is scaling dimension and  $s$  is spin) [10]. In the following, the irreducible representations of infinite conformal algebra are considered. We use the Jacobi identity

$$\begin{aligned} [L_0, [L_n, \mathcal{O}]] &= -[\mathcal{O}, [L_0, L_n]] - [L_n, [\mathcal{O}, L_0]] = n[\mathcal{O}, L_n] + h[L_n, \mathcal{O}] \quad (9) \\ &= (h-n)[L_n, \mathcal{O}] \\ [\bar{L}_0, [\bar{L}_n, \mathcal{O}]] &= -[\mathcal{O}, [\bar{L}_0, \bar{L}_n]] - [\bar{L}_n, [\mathcal{O}, \bar{L}_0]] = n[\mathcal{O}, \bar{L}_n] + \bar{h}[\bar{L}_n, \mathcal{O}] \\ &= (\bar{h}-n)[\bar{L}_n, \mathcal{O}] \end{aligned}$$

The above relations show that  $L_n$  ( $\bar{L}_n$ ) thus lower the value of left (right) conformal weight  $h$  ( $\bar{h}$ ) while  $L_{-n}$  ( $\bar{L}_{-n}$ ) raise it ( $n > 0$ ). We demand that conformal weights is bounded from below, the primary operators are defined by these properties

$$[L_n, \mathcal{O}_p] = 0 \qquad [\bar{L}_n, \mathcal{O}_p] = 0 \qquad n > 0 \qquad (10)$$

By starting with a primary operators  $\mathcal{O}_p$  and using the relation (9), we can build a tower of operators. These operators form an irreducible representation of the conformal algebra.

### 3 Two-point function in free space

We now turn to derive the consequences of conformal invariance to obtain correlation functions. In general, we expect a quasi-primary field  $\mathcal{O}$  to be characterized by its conformal weights  $h$  and  $\bar{h}$  (These fields are invariant under finite sub-group that is generated by sub-algebra  $\{L_{-1}, \bar{L}_{-1}, L_0, \bar{L}_0, L_1, \bar{L}_1\}$ ). We would like to find the form of two-point functions of the conformal invariant operators. Firstly, we find the form of the commutators  $[L_n, \mathcal{O}]$  and  $[\bar{L}_n, \mathcal{O}]$

$$\begin{aligned}
[L_n, \mathcal{O}(z, \bar{z})] &= [L_n, U\mathcal{O}(0)U^{-1}] = [L_n, U]\mathcal{O}(0)U^{-1} + U\mathcal{O}(0)[L_n, U^{-1}] \quad (11) \\
&+ U[L_n, \mathcal{O}(0)]U^{-1} = U\{U^{-1}L_nU - L_n\}\mathcal{O}(0)U^{-1} \\
&+ U\mathcal{O}(0)\{L_n - U^{-1}L_nU\}U^{-1} + \delta_{n,0}h\mathcal{O}(z, \bar{z})
\end{aligned}$$

$U$  is defined in Eq.(7). By using the Hausdorff formula we get

$$\begin{aligned}
U^{-1}L_nU &= e^{-zL_{-1}-\bar{z}\bar{L}_{-1}}L_ne^{zL_{-1}+\bar{z}\bar{L}_{-1}} = e^{-zL_{-1}}L_ne^{zL_{-1}} \quad (12) \\
&= L_n + [L_n, zL_{-1}] + \frac{1}{2!}[[L_n, zL_{-1}], zL_{-1}] + \dots \\
&= \sum_{k=0}^{n+1} \frac{(n+1)!}{(n+1-k)!k!} (z)^k L_{n-k}
\end{aligned}$$

and

$$L'_n = U^{-1}L_nU - L_n = \sum_{k=1}^{n+1} \frac{(n+1)!}{(n+1-k)!k!} (z)^k L_{n-k} \quad (13)$$

where  $z = x + t$  and  $\bar{z} = x - t$ . From above relations, the Eq.(11) gives us

$$\begin{aligned}
[L_n, \mathcal{O}(z, \bar{z})] &= U\{[L'_n, \mathcal{O}(0)] + \delta_{n,0}h\mathcal{O}(0)\}U^{-1} \quad (14) \\
&= z^{n+1}[L_{-1}, \mathcal{O}(z, \bar{z})] + z^n(n+1)U[L_0, \mathcal{O}(0)]U^{-1}
\end{aligned}$$

Now we have  $[L_{-1}, \mathcal{O}] = \partial_z \mathcal{O}$  ( $L_{-1}$  generates  $z$ -translation). Hence we obtain (for  $n \geq -1$ )

$$[L_n, \mathcal{O}(z, \bar{z})] = (z^{n+1}\partial_z + (n+1)hz^n)\mathcal{O} \quad (15)$$

We can exchange  $L_n$  with  $\bar{L}_n$  and using the above equations (11)-(15). We get

$$[\bar{L}_n, \mathcal{O}(z, \bar{z})] = (\bar{z}^{n+1}\partial_{\bar{z}} + (n+1)h\bar{z}^n)\mathcal{O} \quad (16)$$

From equations (15) and (16), we can constrain correlation unctons. We begin by considering two quasi-primary operators  $\mathcal{O}_1(z_1, \bar{z}_1)$  and  $\mathcal{O}_2(z_2, \bar{z}_2)$  which have conformal weights  $(h_1, \bar{h}_1)$  and  $(h_2, \bar{h}_2)$  respectively. Two-point correlation function is defined as

$$G(z_1, z_2; \bar{z}_1, \bar{z}_2) = \langle 0 | \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) | 0 \rangle \quad (17)$$

In this section, we calculated two-point function in the space without boundary condition. We are dealing with quasi-primary fields, so we get four equations which would constrain the form of the correlation function. Invariance under  $z$  and  $\bar{z}$  translations implies

$$\begin{aligned} \langle 0 | [L_{-1}, G] | 0 \rangle &= 0 & \langle 0 | [\bar{L}_{-1}, G] | 0 \rangle &= 0 \\ \Rightarrow \sum_{i=1}^2 \partial_{z_i} G &= 0 & \sum_{i=1}^2 \partial_{\bar{z}_i} G &= 0 \\ \Rightarrow G(z_1, z_2; \bar{z}_1, \bar{z}_2) &= G(z, \bar{z}), & z &= z_1 - z_2, \quad \bar{z} = \bar{z}_1 - \bar{z}_2 \end{aligned} \quad (18)$$

Invariance under dilatation of  $z$  coordinate that generated by  $L_0$  requires

$$\begin{aligned} \langle 0 | [L_0, G(z, \bar{z})] | 0 \rangle &= 0 \\ \Rightarrow \sum_{i=1}^2 (z_i \partial_{z_i} + h_i) G &= 0 \Rightarrow (z_1 \partial_{z_1} + z_2 \partial_{z_2} + h_1 + h_2) G = 0 \\ \Rightarrow (z \partial_z + h) G &= 0 \Rightarrow G(z, \bar{z}) = C(\bar{z}) z^{-h} \quad h = h_1 + h_2 \end{aligned} \quad (19)$$

Similarly, invariance under  $\bar{z}$ -dilatation requires

$$\begin{aligned} \langle 0 | [\bar{L}_0, G(z, \bar{z})] | 0 \rangle &= 0 \\ \Rightarrow \sum_{i=1}^2 (\bar{z}_i \partial_{\bar{z}_i} + \bar{h}_i) G &= 0 \Rightarrow (\bar{z}_1 \partial_{\bar{z}_1} + \bar{z}_2 \partial_{\bar{z}_2} + \bar{h}_1 + \bar{h}_2) G = 0 \\ \Rightarrow (\bar{z} \partial_{\bar{z}} + \bar{h}) G &= 0 \Rightarrow G(z, \bar{z}) = \bar{z}^{-\bar{h}} z^{-h} \quad \bar{h} = \bar{h}_1 + \bar{h}_2 \end{aligned} \quad (20)$$

New information come from requiring SCT invariance which is generated by  $L_1$

$$\begin{aligned} \langle 0 | [L_1, G] | 0 \rangle &= 0 \\ \Rightarrow \sum_{i=1}^2 (z_i^2 \partial_{z_i} + 2h_i z_i) G &= 0 \\ \Rightarrow ((z_1 + z_2) z \partial_z + 2h_1 z_1 + 2h_2 z_2) &= 0 \\ -(h_1 + h_2)(z_1 + z_2) + 2h_1 z_1 + 2h_2 z_2 &= 0 \Rightarrow h_1 = h_2 \end{aligned} \quad (21)$$

and  $\bar{L}_1$

$$\langle 0 | [\bar{L}_1, G] | 0 \rangle = 0 \quad \Rightarrow \quad \bar{h}_1 = \bar{h}_2 \quad (22)$$

The final result is

$$G(z_1, z_2; \bar{z}_1, \bar{z}_2) = \delta_{h_1, h_2} \delta_{\bar{h}_1, \bar{h}_2} (z_1 - z_2)^{-(h_1 + h_2)} (\bar{z}_1 - \bar{z}_2)^{-(\bar{h}_1 + \bar{h}_2)} \quad (23)$$

The above result is two-point function of two conformal invariant fields which live in free space [10].

## 4 Two-point function in semi-infinite space

In this section we calculate two-point function for semi-infinite space with different boundary conditions.

### 4.1 Two-point function in space with boundary condition in surface $\bar{z} = 0$ ( $z = 0$ )

We have a boundary condition in one dimension ( $\bar{z} = 0$ ), so  $\bar{z}$ -translation and  $\bar{z}$ -SCT, which are generated by  $\bar{L}_{-1}$  and  $\bar{L}_1$  respectively, are obviously removed. The reminded symmetry group for this situation is generated by subalgebra  $[L_{-1}, L_1, L_0, \bar{L}_0]$ . (see for example [11] and [12]).

Translation symmetry in  $z$  coordinate, gives us

$$\langle 0 | [L_{-1}, G] | 0 \rangle = 0 \Rightarrow G(z_1, z_2; \bar{z}_1, \bar{z}_2) = G(z; \bar{z}_1, \bar{z}_2) \quad (24)$$

Invariance under  $z$ -dilatation implies

$$\langle 0 | [L_0, G(z, \bar{z}_1, \bar{z}_2)] | 0 \rangle = 0 \Rightarrow G(z, \bar{z}_1, \bar{z}_2) = C_1(\bar{z}_1, \bar{z}_2) z^{-h} \quad (25)$$

and invariance under  $\bar{z}$ -dilatation implies

$$\begin{aligned} \langle 0 | [\bar{L}_0, G(z, \bar{z}_1, \bar{z}_2)] | 0 \rangle &= 0 \\ \Rightarrow (\bar{z}_1 \partial_{\bar{z}_1} + \bar{z}_2 \partial_{\bar{z}_2} + \bar{h}_1 + \bar{h}_2) C_1(\bar{z}_1, \bar{z}_2) &= 0 \\ \Rightarrow C_1(\bar{z}_1, \bar{z}_2) &= \bar{z}_1^{-(\bar{h}_1 + \bar{h}_2)} \Phi\left(\frac{\bar{z}_1}{\bar{z}_2}\right) \end{aligned} \quad (26)$$

where  $\Phi$  is an arbitrary function. Finally, invariance under  $z$ -SCT gives us

$$\langle 0 | [\bar{L}_1, G(z, \bar{z}_1, \bar{z}_2)] | 0 \rangle = 0 \quad \Rightarrow \quad h_1 = h_2 \quad (27)$$

The form of two-point function in semi-infinite space with boundary condition in  $\bar{z}$  coordinate is given by

$$G(z_1, z_2; \bar{z}_1, \bar{z}_2) = \bar{z}_1^{-(\bar{h}_1 + \bar{h}_2)} \Phi\left(\frac{\bar{z}_1}{\bar{z}_2}\right) \delta_{h_1, h_2} (z_1 - z_2)^{-(h_1 + h_2)} \quad (28)$$

We can exchange  $z$  with  $\bar{z}$  and using the above equations (24)-(41), we could obtain

$$G(z_1, z_2; \bar{z}_1, \bar{z}_2) = z_1^{-(h_1 + h_2)} \Phi\left(\frac{z_1}{z_2}\right) \delta_{\bar{h}_1, \bar{h}_2} (\bar{z}_1 - \bar{z}_2)^{-(\bar{h}_1 + \bar{h}_2)} \quad (29)$$

The above relation is two-point function in semi-infinite space with a boundary condition in  $z$  coordinate. We can see, these results (41) and (42) are completely different from result (23).

#### 4.2 Two-point function in space with a boundary condition in surface $z = \bar{z}$

Now we consider a boundary condition on surface  $z = \bar{z}$  i.e.  $(x + t = x - t)$  or  $t = 0$ , so time-translation, Boost and time-SCT are removed. These transformations are generated by  $L_n - \bar{L}_n$  ( $n = -1, 0, 1$ ) generators. In this situation the reminded symmetry group is generated by one copy of non-centrally Virasoro algebra ( $\mathfrak{L}_n = L_n + \bar{L}_n$ ) [6].

$$[\mathfrak{L}_n, \mathfrak{L}_m] = [L_n, L_m] + [\bar{L}_n, \bar{L}_m] = (n - m)(L_{n+m} + \bar{L}_{n+m}) = (n - m)\mathfrak{L}_{n+m} \quad (30)$$

From Eqs.(15), (16) the form of commutator  $[\mathfrak{L}_n, \mathcal{O}]$  is given by

$$[\mathfrak{L}_n, \mathcal{O}] = (z^{n+1} \partial_z + \bar{z}^{n+1} \partial_{\bar{z}} + (n+1)hz^n + (n+1)\bar{h}\bar{z}^n)\mathcal{O} \quad (31)$$

From above equation one can calculate two-point function of two quasiprimary operator  $\mathcal{O}_1$  and  $\mathcal{O}_2$  with conformal weights  $(h_1, \bar{h}_1)$  and  $(h_2, \bar{h}_2)$ . Invariance under  $x$ -translation implies  $(x = \frac{z + \bar{z}}{2})$

$$\langle 0 | [\mathfrak{L}_{-1}, G(z_1, z_2, \bar{z}_1, \bar{z}_2)] | 0 \rangle = 0 \Rightarrow \sum_{i=1}^2 (\partial_{z_i} + \partial_{\bar{z}_i})G = 0 \quad (32)$$



we make the ansatz

$$G(z_1, z_2, \bar{z}_1, \bar{z}_2) = G_1(|z_1 - z_2|) + G_2(|\bar{z}_1 - \bar{z}_2|) + G_3(|z_1 - \bar{z}_2|) + G_4(|\bar{z}_1 - z_2|) \quad (33)$$

Invariance under dilatation which is generated by  $\mathfrak{L}_0$  requires

$$\begin{aligned} < 0 | [\mathfrak{L}_0, G(z_1, z_2, \bar{z}_1, \bar{z}_2)] | 0 > = 0 \\ \Rightarrow \sum_{i=1}^2 (z_i \partial_{z_i} + h_i + \bar{z}_i \partial_{\bar{z}_i} + \bar{h}_i) G = 0 \end{aligned} \quad (34)$$

so

$$\begin{aligned} (z_1 \partial_{z_1} + z_2 \partial_{z_2} + \Delta_1 + \Delta_2) G_1 &= 0 \\ (\bar{z}_1 \partial_{\bar{z}_1} + \bar{z}_2 \partial_{\bar{z}_2} + \Delta_1 + \Delta_2) G_2 &= 0 \\ (z_1 \partial_{z_1} + \bar{z}_2 \partial_{\bar{z}_2} + \Delta_1 + \Delta_2) G_3 &= 0 \\ (\bar{z}_1 \partial_{\bar{z}_1} + z_2 \partial_{z_2} + \Delta_1 + \Delta_2) G_4 &= 0 \end{aligned} \quad (35)$$

$$(36)$$

where  $\Delta_i = h_i + \bar{h}_i$ . Final results for  $G_i$  are

$$\begin{aligned} G_1 &\sim \frac{1}{|z_1 - z_2|^{\Delta_1 + \Delta_2}} & G_2 &\sim \frac{1}{|\bar{z}_1 - \bar{z}_2|^{\Delta_1 + \Delta_2}} \\ G_3 &\sim \frac{1}{|z_1 - \bar{z}_2|^{\Delta_1 + \Delta_2}} & G_4 &\sim \frac{1}{|\bar{z}_1 - z_2|^{\Delta_1 + \Delta_2}} \end{aligned} \quad (37)$$

Spatial-SCT which generated by  $\mathfrak{L}_1$  gives new information,

$$< 0 | [\mathfrak{L}_1, G(z_1, z_2, \bar{z}_1, \bar{z}_2)] | 0 > = 0 \Rightarrow \Delta_1 = \Delta_2 = \Delta \quad (38)$$

Final result of Two-point function in this situation (boundary on surface  $z = \bar{z}$ ) is

$$\begin{aligned} G(z_1, z_2, \bar{z}_1, \bar{z}_2) &= \frac{1}{4} \left( \frac{1}{|z_1 - z_2|^{2\Delta}} \pm \frac{1}{|\bar{z}_1 - \bar{z}_2|^{2\Delta}} \right. \\ &\quad \left. \pm \frac{1}{|z_1 - \bar{z}_2|^{2\Delta}} + \frac{1}{|\bar{z}_1 - z_2|^{2\Delta}} \right) \end{aligned} \quad (39)$$

where the plus and minus signs correspond to Neumann and Dirichlet boundary conditions, respectively. We note that the above result agrees with correlation function of scalar operators has calculated from gravity dual of BCFT [6].

Now, we introduce new variable

$$\zeta = \frac{|z_1 - z_2||\bar{z}_1 - \bar{z}_2|}{|z_1 - \bar{z}_1||z_2 - \bar{z}_2|} \quad (40)$$

Then the two-point function of BCFT may be written in the following form

$$G = \left[ \frac{|z_1 - \bar{z}_1||z_2 - \bar{z}_2|}{|z_1 - z_2||\bar{z}_1 - \bar{z}_2||z_1 - \bar{z}_2||\bar{z}_1 - z_2|} \right]^\Delta F(\zeta) \quad (41)$$

where

$$F(\zeta) = (constant)[(\zeta + 1)^\Delta \pm \zeta^\Delta] \quad (42)$$

The final form of the two-point function (41) agrees with result [8].

## 5 Conclusion

We can use 2d conformal group to constrain correlation functions. Two-point function of conformal invariant fields in free space was found in [10]. We obtained this result by using some methods in conformal field theory. Also by these methods we have computed correlation functions of conformal invariant fields which live in semi-infinite spaces. In this paper we have considered four different case:

1. A system without any boundary condition.
2. A system with a boundary condition on surface  $z = 0$ .
3. A system with a boundary condition on surface  $\bar{z} = 0$ .
4. A system in semi-infinite space with a boundary condition on surface  $z = \bar{z}$  or  $t = 0$ .

The main results of this work are the explicit expressions for two-point functions of conformal invariant fields in semi-infinite spaces as given in Eqs. (41), (42) and (39). The result (39) agrees with the two-point function of scalar operators which was calculated from gravity dual of BCFT [6].

## References

- [1] J. M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113], [arXiv:hep-th/9711200].
- [2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109].

- [3] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].
- [4] T. Takayanagi, Phys.Rev.Lett.107:101602,(2011).
- [5] M. Fujita, T. Takayanagi and E. Tonni, "Aspect of AdS/BCFT," [arXiv:1108.5152[hep-th]]
- [6] M. Alishahiha and R. Fareghbal, "Boundary CFT from Holography," [arXiv:1108.5607 [hep-th]].
- [7] G. W. Gibbons and S. W. Hawking, "Action Integrals and Partition Functions in Quantum Gravity," Phys. Rev. D 15, 2752 (1977).
- [8] J. L. Cardy, Nucl. Phys. B 240 (1984) 514; [arXiv:hep-th/0411189]; D. M. McAvity and H. Osborn, Nucl. Phys. B 455, 522 (1995), [arXiv:cond-mat/9505127].
- [9] A. Bagchi and I. Mandal, "On Representations and Correlation Functions of Galilean Conformal Algebras", Phys. Lett. B 675, 393-397, (2009) [arXiv:0903.4524v2 [hep-th]].
- [10] P. D. Francesco, P. Mathieu, D. Senechal, "Conformal field theory", Verlag New York, (1997).
- [11] M. Henkel "Schrödinger Invariance and Strongly Anisotropic Critical Systems," J.Statist.Phys. 75 (1994) 1023-1061, [arXiv:hep-th/9310081].
- [12] M. R. Setare, V. Kamali "Galilean Conformal Algebra in Semi-Infinite Space ," [arXiv:1101.2339[hep-th]].